They well know, that error correcting codes allow to provide the transmission with a given reliability in satellite and other communication channels at much lower level of a signal in comparison with variant of transmission without coding [1, 2]. For those channels, in which the signal energy is limited, codes create effect of power increasing for the transmitter, that is main technical advantage of coding. For example, the application of the Viterbi algorithm (AV) when code speed R=1/2 and length of the coding register $F=7$ with the “soft” demodulator and binary FM provides at error probability per bit $P_b(e) = 10^{-5}$ code gain (CG) $G=5.2$ dB (diagram 4 at a fig. 1).

Such a CG value appears to be large enough to apply codes actively in real communication systems. But it is much lower than the theoretically achievable bounds. For the mentioned above parameters of coding and the channel the CG is equal 9.4 dB. However further increase CG for real coding systems is connected with fast, and in some cases even exponential growth in complication of signal processing at the receiver. Therefore for many high-speed channels, by which the hard requirements to reliability of reception, the achievement of large values for CG appears to be the extremely difficult and very important task.

At the same time the task of complete usage of power of channels becomes more and more urgent as because of technical and ecological restrictions on energy of signals, and because of finite capacity of selected frequency bands. Thus, though the growth of computing opportunities pawned in the equipment of communication, allows to solve many complex tasks inaccessible earlier, the increasing CG during a long time will be a most important task of communication for many types of channels.

One of the elementary ways of the CG increase is to use multithreshold decoders (MTD) [2,3]. They are updated usual threshold decoders [4], for rather specific codes of length in hundreds and thousand bit, selected by criteria of well known error propagation effect minimization. They give possibility to get the processed message very similar to result of the optimum decoding, but with only linear complexity of decoding algorithm instead of exponential complexity, in particular, for AV. In a correctly constructed MTD for a good code there is possible a repeated improvement of the message, received from the channel, until the MTD decisions become close to the decision of optimal decoder. Thus the number of such attempts of consecutive improvement of the decision of the previous step of decoding can be from three up to hundred and more.

In a fig. 1 the diagram 2 shows error probability per bit $P_b(e)$ as function of the noise level in the BSC (or, the same, signal/noise ratio, dB) for one of convolutional MTD and a code with distance $d=11$ when code speed $R=1/2$. The given result concerns to a case of use of the simple "hard" binary demodulator, which reduces throughput of the channel in comparison with the "soft" modem for decoders AB, almost at 2 dB. Nevertheless, MTD appeared to be more effective then AB. Let's emphasize, that it is possible because of the almost optimum decision MTD for much longer code, than the code of length $K=7$, which was taken for an estimation of opportunities AB. The diagram 1 shows error probability per bit $P_b(e)$ for code with distance $d=9$ and diagram 3 for $d=13$. They all are both theoretical and experimentally imitated for MTD.
For simultaneous comparison of decoding algorithms and their CG the dotted lines in a
fig.1 specify levels of equal CG for codes with $R=1/2$: 4, 6 and 8 dB. They help to compare codes
with respect to the main criterion of efficiency of their application.

For illustration of the main performance with identical conditions of work with "soft"
modems the code with code rate $R = 1/2$ for "soft" MTD was taken. The code had rather large length
and minimal code distance $d=9$, and its decoding practically coincided with optimum one, when the
value of signal/noise ratio in the channel was $-1.0$ dB, as follows from the diagram m9. It shows
both theoretical estimations and experimental results for MTD. The same results for codes with $d=7$
and $d=11$ show lines m7 and m11.

Proceeding from widely known estimations and submitted results, it is possible to consider,
that MTD provides the almost theoretically best characteristics of CG for high level of noise. Let's
note, that if the choice of the more longer codes is carried out and they are decoding with AV, the
growth of efficiency appears not so significant, as it would be possible to expect. This problem was
discussed with details in [6] In particular, code with $K=11$ [1, the fig. 12.13] for AV at $R=1/2$ is
characterized by the diagram 5, and the opportunities of a code with $K=15$ are given on the diagram 6.
They show the limited growth of efficiency of this algorithm even in case of growth of volume of AV
calculations in comparison with $K=7$ by many decimal orders.

Let's notice, that increase of efficiency MTD appears significant in comparison and with
usual threshold decoding, and with AV owing to such organization in procedures of correction, when
for convolutional variant of realization of coding the delay of decoding becomes rather significant,
achieving thousand and even many tens thousand code symbols because of the large growth of length
of codes and significant number of iterations of decoding. But it is necessary to emphasize, that from
the theoretical point of view the high efficiency is provided only really with very long codes. In a fig.1
the lowest estimations of bit error probability for block codes are submitted for $R=1/2$ in usual BSC
without memory: from $n=1000$ up to $n=10000$ in the assumption, that they were decoded in an
optimum way, that is unattainable. The diagrams 7,8 and 9 again correspond to codes with $K=7,11$
and 15, decoded with AV, but in BSC. This modeling was fulfilled with the “hard" modem, when
with equal conditions the potential opportunities of codes, instead of algorithms, are estimated.

The comparison of the basic competing algorithms for satellite and space communication in
[2] and in the given report shows, that among based error correction algorithms preferable are AV
for short codes and MTD. In BSC MTD has no any competitors, as a method of almost optimal
decoding with a simple realization. If the large delay of acceptance of the decision is not
restriction for application, MTD appears much faster than AB, much more effective and a very
simple too.

We must note also, that specified in [5] by Berlecamp the profit of additional CG as
$1’000’000$ per each dB, is now much large because of deficiency of a free frequencies and set of
technical and ecological restrictive problems. It means, that the decoding algorithms and further will
be very highly appreciated in terms of CG.

As it is known, on the basis of the main algorithms it is possible to build and concatenated
codes, which even more raise efficiency of application of coding. They must be discussed in special
issues. But it seems to be obvious, that the concatenated circuits always appear more effective, if
higher results provide codes and algorithms of decoding used in each stage of the appropriate code
design. From here it is possible to make enough real assumption that MTD in concatenated circuits
will supply higher characteristics, than other concatenated approaches. It is necessary to note also, that
concatenation idea was always connected with rather long codes. But it is just a condition to
organize effective decoding on the basis of MTD.

Thus, the data, submitted in the report, allow to specify new opportunities of realization of
decoding algorithms similar to MTD, which looks almost like optimal one in a wide range of
parameters of channels of satellite and space communication. The MTD creation for real systems has
already proved achievement of the predicted characteristics and declared opportunities for block, convolutional, parallel, concatenated and iterated codes, when used for both binary, multipositional and others nonbinary signals.

For additional information you can ask author in Russia, Moscow:

**Space Research Institute**: +7 095 333-23-56, +7 095 333 23 56;
**NIIRadio**: +7 095 261 03 27, +7 095 261 54-44;
Home tel.: +7 095 573 57 32, mobile +7 916 18 86 28 or by e-mail zolotasd@yandex.ru (or zolotarev@iki.rssi.ru as reserved) (in Russia).

**Literature**

Вероятности ошибки алгоритмов декодирования (BER for decoders)