ABSTRACT. In this article several ways of improving performance of multithreshold decoder are considered. Those ways are based on the usage of multithreshold decoder as a part of different coding schemes. All the methods offered allow, with a little complication, to improve the performance of multithreshold decoder more than on 1 dB at bit error rate of $10^{-5}$.

INTRODUCTION. In the theory coding some methods for coding and decoding which allow to work near of channel capacity are known [1…4]. The special place among them is occupied with multithreshold decoders (MTD) [4…10], providing almost optimum decoding even for very long self-orthogonal codes (SOC) with linear complexity.

The diagram for MTD with two decoding iterations of convolutional SOC with code rate $R=1/2$, code distance $d=5$, length of code restriction $n_A=14$ is shown in fig. 1. It’s seen that MTD is the development of the ordinary threshold decoder [11]. The work of MTD is based on iterative decoding principle when decoder repeatedly tries correct received information symbols. It should be note that MTD saves simplicity and speed of threshold decoder as each iteration of MTD differs from threshold decoder only presence “difference” register in which the information symbols changed by threshold element are marked.

Bit error performance is shown in fig. 2 over a binary symmetric channel (BSC) for several SOC with code distance $d=9$, chosen according to criterion of minimization of error propagation effect. At data acquisition of dependences it was used from 5 up to 15 decoding iterations. For comparison in figure dashed lines submit bit error rate of an optimum decoder for these codes. As follows from the
submitted curves, application MTD for decoding codes with small error propagation provides almost optimum decoding that allows to get coding gain more than 5 dB at bit error rate $P_b=10^{-5}$ over a BSC.
Characteristics MTD over a channel with additive white gaussian noise (AWGN) for the same codes are shown in fig. 3. Apparently, and in this case the decision of an optimum decoder is practically achieved. Thus, transition to soft-decision modem allows to increase coding gain on 1.5..2 dB in comparison with application only hard-decision modem. We shall notice that such results are unattainable at use of optimum Viterby decoder [12] because of its complexity is grown exponentially with constructive length of a code.

![Figure 3](image)

Unfortunately, MTD it is possible to apply only to codes with small code distance \( d \) (about 10..16). In this connection there is inconvenient an increase of coding gain with application of longer codes.

Among possible approaches to improvement of MTD’s coding gain performance it is necessary to allocate its use in some concatenated codes, such as parallel codes [13], codes with non-uniform power and codes with the selected branches. Though separate use of each of the given approaches can increase coding
gain only on 0.3..0.7 dB, joint application of these schemes allows to get significant better results.

**CONCATENATING OF SELF-ORTHOGONAL CODES DECODING WITH MULTITHRESHOLD DECODER AND PARITY CHECK CODES.** The special place among concatenated codes based on MTD occupies its cascading with parity check codes (PCC) [14] which use allows increasing efficiency of coding application significantly. Feature of this scheme consists that such cascading practically does not demand additional expenses for the equipment (in the circuit of coding it is required to add only one adder on the module 2) whereas use in a cascade code, for example, Reed-Solomon code incomparably is more complex.

Let's consider the decoder of concatenated code which external code is parity check code and internal – a self-orthogonal code decoding with MTD. The length $n_1$ of parity check code is necessary for choosing enough big (about 25..100) that losses in power because of reduction of the common code rate were insignificant.

At the first stage of job of this decoder as in any concatenated code decoder of the internal code (MTD) carries out decoding sequence received from the channel. Let after the last iteration MTD remembers all sums of checks concerning all decoded symbols. Then for correction of single errors by parity check code for all $n_1$ symbols of its code block reliability of decision $\Delta_i=|m_i-T|$ is calculated. Here $m_i$ – the sum on the threshold element while decoding $i$–th symbol, $T$ – value of threshold on the threshold element of the last decoding iteration, $i=1..n_1$. Then, in case of detection error by parity check code (i.e. in case of the sum of bits in code block of PCC on the module 2 differs from zero), a symbol with minimal reliability $\Delta_i$ is corrected. If there are some symbols with the minimal reliability change of information symbols is not made.

In fig. 4 the upper bound for bit error rate of concatenated code over a BSC are shown. This code consist of SOC with $d=7, 9, 11$ (it is hereinafter considered, that this SOC are decoded with MTD) and parity check code with length $n_1=25$. The technique
of data acquisition of ratings is in detail considered in [14]. In the same figure modeling results for a concatenated code based on MTD for SOC with $R=1/2$, $d=7$ and $d=9$ are shown. We shall notice, that use simple parity check code together with SOC has allowed to get an additional coding gain about 1..1.5 dB at bit error rate $P_b=10^{-5}$.

The bit error performance of the concatenated codes consisting of same SOC and PCC with $n_1=50$ over an AWGN channel is shown in fig. 5. Apparently, and in this case the cascade code appears much better not cascade. It is necessary to note, that at reception of submitted curves parity check code it was used on several decoding iterations, thus as though "helping" to MTD at decoding internal SOC. Also we shall note, that considerably more complex concatenated code consisting of a Reed-Solomon code (255, 223, 33) and convolutional code with length of code restriction $K=7$ and code rate $R=1/2$ decoding with optimum Viterby decoder, even at smaller code rate ($R=0.437$) concedes to the concatenated code based on MTD at $P_b>10^{-6}$. We shall notice, that the considered way of cascading allows to improve characteristics MTD only in the field of its effective work.

![Figure 4](image-url)
APPLICATION MTD IN SCHEMES WITH PARALLEL CODING. For approach field of MTD’s effective work near to channel capacity MTD can be used in earlier mentioned schemes with parallel coding [13]. In a basis of this construction allocation in SOC $C_0$ with code distance $d_0$ and code rate $R_0$ of some constituent code $C_1$ with code rate $R_1 > R_0$, too being SOC is lays. The code distance $d_1$ of the constituent code gets out considerably smaller $d_0$, and, hence, the field of its effective work will be closer to Shannon limit. First at decoding a parallel code some decoding iterations of the constituent code $C_1$ are carried out, allowing approximately in 10 or more times to lower bit error rate in the information sequence received from the channel. Then decoding of the code $C_0$ is performed. Distinctive feature of the given scheme is that here the external code works with code rate $R_0$ while in usual concatenated codes code rate of an external code is close to unit. The given property provides essential advantage MTD over other concatenated codes.

For an example on fig. 6 and 7 modeling results of codes with parallel coding over a BSC and an AWGN channel for SOC with $R_0=6/12$, $d_0=13$ and $R_0=5/10$, $d_0=15$
are shown. In a parallel code with $d_0=13$ the constituent code with $R_1=6/11$, $d_1=7$ in this case has been selected, and in a code with $d_0=15$ the code with $R_1=5/9$, $d_1=9$ has been selected. Curves labeled “constituent” in this figure show bit error rate for constituent codes of the parallel schemes. Dashed lines without markers in the given figures show bit error rate for optimum decoding of SOC with $d=7$, 9, 11, 13 and 15. For comparison on fig. 6 and 7 characteristics of ordinary SOC decoding with MTD with similar $d$ and $R$ also are shown.

![Figure 6](image)

As follows from the analysis of the submitted dependences, application of parallel coding allows to approach field of effective work of MTD to channel capacity approximately on 0.5 dB.

**CONCLUSION.** In paper two methods of MTD’s efficiency improvement have been considered. First of them consists in addition to SOC decoding with MTD external parity check code, that allows to reduce bit error rate in the field of decoder’s effective work in 10 or even more times. The second method will consist in allocation in usual SOC some constituent code used in the decoder as internal code. The given
way named parallel coding, allows to approach area of effective work of MTD to channel capacity approximately on 0.5 dB. It is necessary to note, that the submitted coding schemes possess almost same complexity of practical realization, as usual MTD [4], and, hence, appear essentially easier known methods of decoding at comparable efficiency. Taking into account the aforesaid, the methods of error correction considered in the paper can be recommended to application in perspective high-speed digital communication systems.

The additional information about MTD can be found on web-site [15].

The bibliographic list


